NATIONAL INSTITUTE OF TECHNOLOGY WARANGAL

Department of Mathematics

DYNAMICAL SYSTEMS

Problem Sheet-I: Dynamical System Analysis

Course: Dynamical Systems

Instructor: Dr. Sivalingam S M

Problem Sheet: I

Student Name: _____

Roll Number: _____

Instructions: Each student has been assigned a different two-dimensional autonomous dynamical system. Analyse your assigned system by answering all the questions below. Show the mathematical steps clearly.

1. Write your assigned dynamical system clearly and identify the following:
   (a) the state variables
   (b) the dimension of the system
   (c) whether the system is autonomous or non-autonomous
   (d) all parameters appearing in the system
   (e) any assumptions imposed on the parameters.

   If the system can be obtained from a higher-order differential equation, explain how the corresponding first-order system is obtained, or if the system is in first-order, convert it to a higher-order system.
2. Determine all equilibrium points
3. Construct the Jacobian matrix and evaluate at every equilibrium point and determine the eigenvalues.
4. Classify the equilibrium point based on the analysis of the trace, determinant, and discriminant (For each equilibrium, provide a mathematical justification for your classification. Do not simply state the classification.)
5. Construct the phase portrait (Hint: Choose at least 2-3 different initial conditions. Numerically solve the system for these initial conditions and plot the corresponding trajectories in the phase plane. If an exact solution of the system can be obtained in closed form, obtain it.)
6. Based on the analytical calculations, explain the observed behaviour,
   - Why do trajectories approach or move away from particular equilibria?
   - Why do some trajectories spiral?
   - Why do some trajectories move along particular directions?
   - Are there regions with qualitatively different behaviour?

   Your explanation must connect the phase portrait to the mathematical analysis of the system.

7. Calculate and interpret the divergence.
8. Investigate whether the system possesses periodic solutions and interpret it.
9. Investigate whether the system possesses a limit cycle. (use Bendixson's criteria). Explain if it is a periodic attractor or not?
10. Investigate the behaviour of the states of the system as $t \to \infty$?

Submission: Submit the mathematical analysis in LaTeX, and the resulting PDF.

Problem Sheet-I: Dynamical Systems

Each student must perform the complete qualitative analysis of the dynamical system assigned below.

No.	Roll No.	Assigned Dynamical System
1	25MAC1R01	$\dot{x} = -x + 2y, \quad \dot{y} = -3x - 2y$
2	25MAC1R02	$\dot{x} = 2x + y, \quad \dot{y} = x + 3y$
3	25MAC1R03	$\dot{x} = 2x + y, \quad \dot{y} = 3x - y$
4	25MAC1R04	$\dot{x} = -2x + y, \quad \dot{y} = -x - 3y$
5	25MAC1R05	$\dot{x} = x - 2y, \quad \dot{y} = 2x + y$
6	25MAC1R06	$\dot{x} = y, \quad \dot{y} = -4x$
7	25MAC1R07	$\dot{x} = -x + y, \quad \dot{y} = -x - y$
8	25MAC1R09	$\dot{x} = -x, \quad \dot{y} = -x - y$
9	25MAC1R10	$\dot{x} = -2x, \quad \dot{y} = -2y$
10	25MAC1R11	$\dot{x} = x + y, \quad \dot{y} = -y$
11	25MAC1R12	$\dot{x} = y, \quad \dot{y} = -x - x^3$
12	25MAC1R13	$\dot{x} = y, \quad \dot{y} = -x + x^3$
13	25MAC1R15	$\dot{x} = y, \quad \dot{y} = x - x^3 - y$
14	25MAC1R16	$\dot{x} = y, \quad \dot{y} = 2x - x^3 - y$
15	25MAC1R17	$\dot{x} = y, \quad \dot{y} = -x + x^3 - y$
16	25MAC1R18	$\dot{x} = y, \quad \dot{y} = -x - 0.5y - x^3$
17	25MAC1R19	$\dot{x} = y, \quad \dot{y} = x - x^3 - 0.4y$
18	25MAC1R21	$\dot{x} = y, \quad \dot{y} = -2x + x^3 - 0.6y$
19	25MAC1R22	$\dot{x} = y, \quad \dot{y} = -x + x^3 - 0.2y$
20	25MAC1R23	$\dot{x} = y, \quad \dot{y} = -x - 0.8y + x^3$
21	25MAC1R24	$\dot{x} = y, \quad \dot{y} = (1 - x^2)y - x$
22	25MAC1R25	$\dot{x} = y, \quad \dot{y} = (1 - x^2)y - 2x$
23	25MAC1R26	$\dot{x} = y, \quad \dot{y} = 0.5(1 - x^2)y - x$
24	25MAC1R27	$\dot{x} = y, \quad \dot{y} = (1 - x^4)y - x$
25	25MAC1R28	$\dot{x} = y, \quad \dot{y} = (1 - 2x^2)y - x$
26	25MAC1R29	$\dot{x} = y, \quad \dot{y} = (1 - x^2)y - x - x^3$
27	25MAC1R30	$\dot{x} = y, \quad \dot{y} = (1 - x^2)y - 0.5x$

No.	Roll No.	Assigned Dynamical System
28	21MAE0006	$\dot{x} = x(1-x) - 0.5xy, \quad \dot{y} = -0.4y + 0.3xy$
29	22MAE0003	$\dot{x} = x(1-x) - xy, \quad \dot{y} = -0.5y + 0.8xy$
30	22MAE0004	$\dot{x} = x(1-x/4) - 0.6xy, \quad \dot{y} = -0.3y + 0.4xy$
31	22MAE0007	$\dot{x} = x(1-x) - 0.8xy, \quad \dot{y} = -0.2y + 0.5xy$
32	22MAE0008	$\dot{x} = 1.2x - x^2 - xy, \quad \dot{y} = -0.7y + 0.6xy$
33	22MAE0009	$\dot{x} = x - 0.5x^2 - xy, \quad \dot{y} = -0.4y + 0.7xy$
34	22MAE0010	$\dot{x} = x(1-x) - 0.4xy, \quad \dot{y} = -0.6y + 0.9xy$
35	22MAE0011	$\dot{x} = x(1-x/5) - 0.7xy, \quad \dot{y} = -0.5y + 0.4xy$
36	22MAE0012	$\dot{x} = 1 - x - xy, \quad \dot{y} = x - y^2$
37	22MAE0013	$\dot{x} = 1 - 2x + xy, \quad \dot{y} = x - y - xy$
38	22MAE0015	$\dot{x} = 2 - x + xy, \quad \dot{y} = x - y^2$
39	22MAE0016	$\dot{x} = 1 - x + x^2y, \quad \dot{y} = x - x^2y - y$
40	22MAE0017	$\dot{x} = x(1-x) - xy, \quad \dot{y} = y(1-y) + 0.5xy$
41	22MAE0018	$\dot{x} = x(1-x) + 0.4xy, \quad \dot{y} = y(1-y) - 0.6xy$
42	22MAE0019	$\dot{x} = y, \quad \dot{y} = -\sin x - 0.3y$

For the Ph.D Scholar:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= \mu(1-x^2)y - x + x^3 - \alpha x^5, \end{aligned} \qquad \mu > 0, \quad \alpha > 0.$$